P P SAVANI UNIVERSITY

Third Semester of B. Tech. Examination

Nov-Dec 2021

SESH2070 Mathematical Methods for Machine Learning

03.12.2021, Friday **Time: 09:00 a.m. To 11:30 a.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 Answer the Following **[05]**

(i) Find the integrating factor of the differential equation $\frac{dy}{dx}-\left(\frac{1}{x}\right)y=x^2$

(ii) What is the general solution of $\left(D^2-D+1\right)y=0$

(iii) Find the complete integral for $z=px+qy+p^2q^2$

(iv) Find complementary function for partial differential equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=0$

(v) Evaluate $\int_{-1}^{1} x|x|\,dx$

Q - 2 (a) Solve $\left(2x\cos y+3x^2y\right)dx+\left(x^3-x^2\sin y-y\right)dy=0$, $y(0)=1$ **[05]**

Q - 2 (b) Solve $y''-4y'+4y=\frac{e^{2x}}{x}$ using variation of parameters method. **[05]**

OR

Q - 2 (a) Solve $\frac{dy}{dx}+(\tan x)y=\sin 2x$, $y(0)=0$ **[05]**

Q - 2 (b) Solve $y''-2y'+5y=25x^2+12$ using method of undetermined coefficients. **[05]**

Q - 3 (a) Solve $\left(D^2-2DD'\right)z=\sin x\cos 2y$ **[05]**

Q - 3 (b) Solve using Lagrange's method $x(y-z)p+y(z-x)q=z(x-y)$ **[05]**

OR

Q - 3 (a) (1) Solve $p+q=pq$ **[05]**

(2) Solve $p+q=\sin x+\sin y$

Q - 3 (b) (1) Form partial differential equation from relation $f\left(x^2+y^2, z-xy\right)=0$ **[05]**

(2) Solve using Lagrange's method $yzp-xzq=xy$

Q - 4 Attempt any one **[05]**

(i) Find Fourier series of $f(x)=\frac{\pi-x}{2}$, $0<x<2\pi$

(ii) Find Fourier series of $f(x)=x+|x|$, $-\pi<x<\pi$

SECTION – II

Q - 1 Answer the Following **[05]**

(i) Evaluate $\int_{-\pi}^{\pi} |x| \sin x \, dx$

(ii) Define gradient of a scalar function.

(iii) Define curl of a vector point function.

(iv) If $\operatorname{div} \overline{F} = 0$ then $\overline{F}$ is called __________.

(v) Define an odd function with examples.

Q - 2 (a) Find the unit vector normal to the surface $x^2 + y^2 - z = 10$ at the point $(1,1,1)$. **[03]**

Q - 2 (b) **[07]**

(1) If $f = \tan^{-1}\left(\frac{y}{x}\right)$, find $\operatorname{div}(\nabla f)$

(2) If $f = 2x^2 - 3y^2 + 4z^2$, find $\operatorname{curl}(\nabla f)$

OR

Q - 2 (a) Determine the constants a and b such that curl of $(2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} + (3xy + 2byz)\hat{k}$ is zero. **[05]**

Q - 2 (b) Find the directional derivative of $f(x,y,z) = xy^2 + yz^2$ at the point $(1,1,1)$ in the direction of the vector $\overline{a} = 3\hat{i} + 4\hat{j} + 2\hat{k}$ **[05]**

Q - 3 (a) If $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ then show that **[07]**

(1) $\int_c \overline{F}.\overline{dr}$ is independent of path.

(2) Find its scalar potential function ϕ such that $\overline{F} = \nabla \phi$

Q - 3 (b) If $\overline{F} = (ax^2y - yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal then find value of a. **[03]**

OR

Q - 3 (a) Verify Green's theorem in plane for $\int_c \left[x^2dx - xydy\right]$ where C is the boundary of the region bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = a$ **[07]**

Q - 3 (b) If $\overline{F} = x^2\hat{i} + xy^2\hat{j}$ then evaluate $\int_c \overline{F}.\overline{dr}$ from $(0,0)$ to $(1,1)$ along the path $y = x$ **[03]**

Q - 4 Attempt any one **[05]**

(i) Find half range cosine series of $f(x) = \pi - x$, $0 < x < \pi$

(ii) Find Fourier series of $f(x) = x^2 - 2$, $-2 < x < 2$

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